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Frontier, )

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TSPV

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*TE*

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*AE*

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λ. Koopman & Deberio  
Υ. Technical Efficiency  
ϒ. Allocative Efficiency

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$$(Y) \quad (X, X_2)$$

$$F(\alpha x_1, \alpha x_2) = \alpha F(x_1, x_2) \quad (\text{CRS})$$

$$F(\alpha x_1, \alpha x_2) > \alpha F(x_1, x_2) \quad (\text{IRS})$$

$$F(\alpha x_1, \alpha x_2) < \alpha F(x_1, x_2) \quad (\text{DRS})$$

(CRS)

$$( \quad )$$

$$(y = y_0)$$

$x_2, x_1$

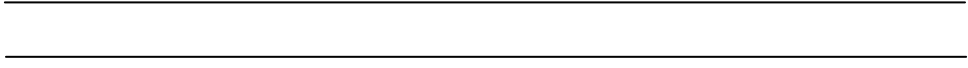
$$: (y = y_0)$$

$$y = F(x_1, x_2) \Rightarrow F_1 dx_1 + F_2 dx_2 = 0 \Rightarrow MRTS = \frac{dx_2}{dx_1} = -\frac{F_1}{F_2}$$

L K

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1 . Return to Scale  
 2 . Isoquant

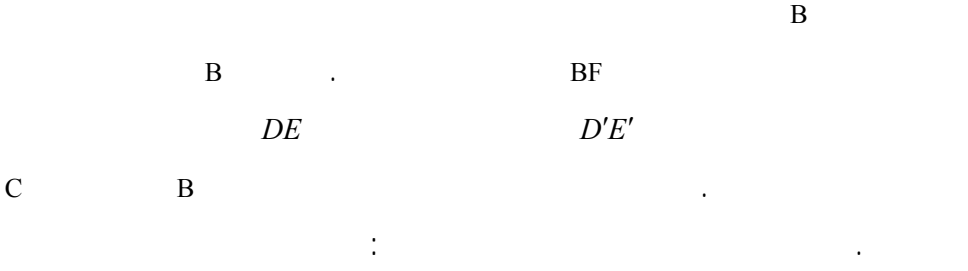


BF B . I DE  
A (B )  
A (BA ) B  
:  $\frac{OB}{OA}$  . C  
A :  $TE = \frac{OB}{OA}$   
B :  $\frac{OF}{OB}$

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$$EE = TE_i \times AE_i = \frac{OB}{OA} \times \frac{OF}{OB} = \frac{OF}{OA}$$

)  $SS'$  ( )  
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 -  
 (A, B, C, D)

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$$Y_j = \sum_{i=1}^n \alpha_i X_{ij} + \varepsilon_j$$

$i$	$X_{ij}$	$j$	$Y_j$	
$i$	$\alpha_i$	$j$	$\varepsilon_j$	$j$
			$\varepsilon_j$	

$$\sum_{i=1}^n \alpha_i x_{ij} - y_j = -\varepsilon_j \quad , \quad \bar{x}_i = \frac{1}{n} \sum X_{ij}$$

$$\min \quad \alpha_0 + \alpha_1 \bar{x}_1 + \alpha_2 \bar{x}_2 + \dots + \alpha_m \bar{x}_m$$

$$\begin{cases} \alpha_0 + \alpha_1 x_{11} + \alpha_2 x_{21} + \dots + \alpha_m x_{m1} \geq y_1 \\ \alpha_0 + \alpha_1 x_{1n} + \alpha_2 x_{2n} + \dots + \alpha_m x_{mn} \geq y_n \end{cases}$$

$n$	$x_1, \dots, x_m$	$\bar{x}_1, \dots, \bar{x}_m$
$:$		

$$\hat{Y}^* = \hat{\alpha}_0 + \hat{\alpha}_1 X_{1j} + \hat{\alpha}_2 X_{2j} + \dots + \hat{\alpha}_m X_{mj}$$

$$TE_j = \frac{y_j}{\hat{y}_j} \times 100 \quad , \quad j = 1, \dots, n$$



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*TE*

$$\text{Min} \quad \sum_{i=1}^n [y_i - f_i(x_1, x_2, \dots, x_n, \beta)]^2,$$

$$\text{s.t} \quad y_i \leq f(x_1, x_2, \dots, x_n, \beta)$$

*K L*

*TE*

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$$\text{Ln} y_i = \text{Ln} f(x_i, \beta) + e_i, e_i \geq 0$$

$$\beta ( \quad ) x_i \quad i \quad y_i$$

$e_i$

*TP*

*TP*

*TP*

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$e_i$

$$e = u_i - v_i$$

$u_i$

$u_i$

$v_i \geq 0$

$v_i$

$\delta_u^r$

$$\text{Ln}y_i = \text{Ln}A + \alpha_1 \text{Ln}X_{i1} + \dots + \alpha_k \text{Ln}X_{ki} + e_i, \quad e_i = u_i - v_i$$

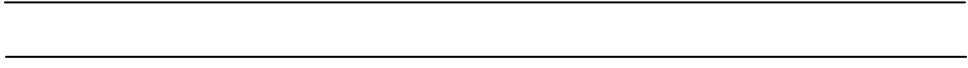
$e_i$

(MLE)

$$\text{LnTE} - V_i = \text{Ln}y_i - \text{Ln}\hat{y}_i, \quad \text{TE} = \frac{y_i}{\hat{y}_i}$$

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1. White Noise



$$TC = (y, w, z, t) + e_i, \quad e_i = u_i - v_i$$
$$t = \left( \frac{v_i}{\delta_u} \right) z + w y$$
$$v_i = \delta_u \left( \frac{u_i}{v_i} \right) \delta_u$$

$$V_i = \gamma z_i + w_i$$
$$\delta_w$$

$$+ \left( \frac{v_i}{\delta_u} \right) + \left( \frac{u_i}{v_i} \right)$$
$$+ +$$

$$LAC = \left( \frac{v_i}{\delta_u} \right) \left( \frac{u_i}{v_i} \right) \left( \frac{u_i}{v_i} \right)$$

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$$\begin{aligned}
 LnTc &= \beta_0 + \beta_1 \sum_{i=1}^Y Lny_{ist} + \beta_2 \sum_{i=1}^Y Lnw_{ist} + \gamma_{st} T_{st} + \frac{1}{Y} \beta \sum (Lny_{ist})^Y + \\
 &\quad \frac{1}{Y} \beta (Lnw_{ist})^Y + \frac{1}{Y} Q_{st} T_{st}^Y + \beta_{ii} \sum_{i=1}^Y \sum_{i=1}^Y Lny_{ist} Lnw_{ist} + \beta_i Q_{ist} \\
 &\quad \sum_{i=1}^Y Lny_{ist} T_{st} + \beta_i \gamma_{st} \sum_{i=1}^Y Lnw_{ist} + T_{st} + Q_i \sum_{i=1}^Y Lnz_{ist} + u_{it} + v_{it} \\
 &\hspace{30em} t,s,i \\
 &\hspace{30em} + \hspace{30em} + \hspace{30em} : TC \\
 &\hspace{30em} : y_1 \\
 &\hspace{30em} : y_Y \\
 &\hspace{30em} ( \hspace{30em} ) : w_1 \\
 &\hspace{30em} : w_Y \\
 &\hspace{30em} T = \{1, 2, \dots, Y\} : T \\
 &\hspace{30em} u_{it}, v_{it}
 \end{aligned}$$

$$( \hspace{2em} + \hspace{2em} + \hspace{2em} + \hspace{2em} )$$

T

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*Frontier<sup>γ, λ</sup>*

*LR*

: λ

$$\lambda = -\gamma \{ \ln[L(H_{i,t})/L(H_{i,t-1})] \} = -\gamma \{ \ln[L(H_{i,t})] - \ln[L(H_{i,t-1})] \}$$

*L(H<sub>i,t</sub>), L(H<sub>i,t-1</sub>)*

*x<sup>γ</sup>* λ *H<sub>i,t</sub>* · *H<sub>i,t-1</sub>* *H<sub>i,t</sub>*

*H<sub>i,t</sub>* *H<sub>i,t-1</sub>*

*v<sub>i</sub>*

:

: *v<sub>it</sub>* ( ) : *I*

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γ . Technical Change  
 γ . Like Lihood Ratio

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$$v_{it} = v_i \{EXP[-\eta(t - T)]\}$$

$H_0 : \eta = \dots$

$H_0 : \eta = \dots$

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$: H$

$H_0 : \theta = \dots$

$\theta = \dots$

$H_0 : \theta = \dots$

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$$v_{it} \approx N(m_{it}, \delta_v^2)$$

$$v_{it} = \theta + \theta_{its} \sum_{i=1}^y Z_{its}$$

$Z_i$

$Z$

$Z_1$

$Z_1$

...

$H$

$H, I$

(I)

$\eta$

$I$

$(\eta = - / )$

$t$				
/	/	/		$B_{\cdot}$
/	/	/	$\log ( \quad )$	$B_{\vee}$
- /	/	- /	$\log ( \quad )$	$B_{\nabla}$
/	/	/	$\log ( \quad )$	$B_{\nabla}$
/	/	/	$\log ( \quad )$	$B_{\nabla}$
- /	/	- /		$B_{\Delta}$
/	/	- /	$\log ( \quad ) * \log ( \quad )$	$B_{\nabla}$
- /	/	- /	$\log ( \quad ) * \log ( \quad )$	$B_{\nabla}$
/	/	/	$\log ( \quad ) * \log ( \quad )$	$B_{\wedge}$
- /	/	- /	$) * \log ( \quad )$	$B_{\natural}$
			$\log ( \quad )$	
- /	/	/	$( \quad )$	$B_{\vee}$
- /	/	- /	$) * \log ( \quad )$	$B_{\vee}$
			$\log ( \quad )$	
/	/	- /	$\log ( \quad ) * \log ( \quad )$	$B_{\nabla}$
/	/	/	$) * \log ( \quad )$	$B_{\nabla}$
			$\log ( \quad )$	

...

- /	/	- /	$\log ( \quad )^* \log ( \quad )$	$B_{\gamma \tau}$
- /	/	- /	$\log ( \quad )^* \log ( \quad )$	$B_{\gamma \delta}$
/	/	/	$\log ( \quad )^* \log ( \quad )$	$B_{\gamma \tau}$
- /	/	- /	$\log ( \quad )^* ( \quad )$	$B_{\gamma \nu}$
/	/	/	$\log ( \quad )^* ( \quad )$	$B_{\gamma \lambda}$
- /	/	- /	$\log ( \quad )^* ( \quad )$	$B_{\gamma \eta}$
/	/	/	$\log ( \quad )^* ( \quad )$	$B_{\gamma \cdot}$

		Log like lihood	$\lambda$	$X^\gamma$	
$H_0$	$\beta_i^\wedge = \beta_{ii} = \beta_i \gamma_{st} = \cdot$ $i = \delta$	/	/	/	
$H_1$	$\beta_i^\vee \neq \beta_{ii} \neq \beta_i \gamma_{st} \neq \cdot$ $i = \lambda$	/			

/



		Log like lihood	$\lambda$	$X^r$	
$H_0$	$\gamma_{st} = \theta_{st} = \beta_i^r \gamma_{st} = \cdot$ $i = 1$	/	/	/	
$H_1$	$\gamma_{st} \neq \theta_{st} \neq \beta_i^r \gamma_{st} \neq \cdot$ $i = 1$	/			

/

$$\beta_1 = \beta_2 = \beta_3 = \beta_4 = \cdot$$

		Log like lihood	$\lambda$	$X^r$	
$H_0$	$\beta_i = \cdot \quad i = 1, \dots, r$	/	/	/	
$H_1$	$\beta_i \neq \cdot$	/			

		Log like lihood	$\lambda$	$X^r$	
$H_0$	$\mu = \cdot$	/	/	/	
$H_1$	$\mu \neq \cdot$	/			

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λ . Elasticity  
Υ . Return to Scale

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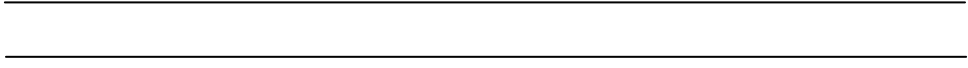
*TSP.V*

*t*

$$V_i = f(T, D_\lambda, D_\gamma) \quad T = \quad D_\lambda = \quad D_\gamma = ( \quad = \quad )$$

		( ) <i>t</i>
<i>C</i> =	+ /	/
<i>T</i> =	+ /	/
<i>D<sub>λ</sub></i>	+ /	/
<i>D<sub>γ</sub></i>	- /	- /

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